MAR Plus For Electrical Engineering

1. Ohm's and Kirchhoff's laws
2. Electrical Network
3. Analysis of Electrical Circuits
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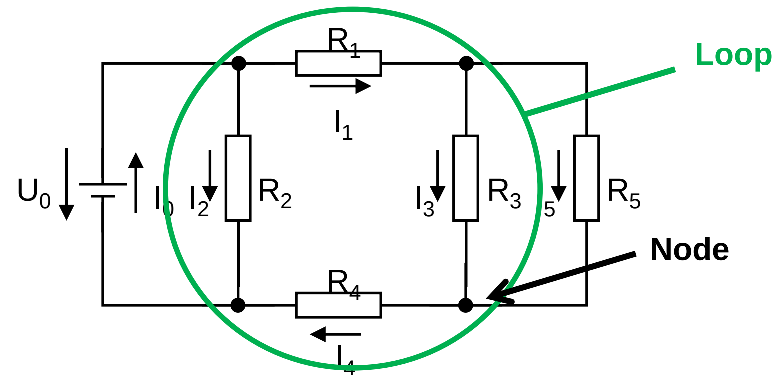
Ohm’s and Kirchhoff’s laws



Ohm’s Law

The current in a circuit is directly proportional to the voltage and inversely proportional to the resistance:

$$I[A] = \frac{U[V]}{R[\Omega]}$$

Kirchhoff’s First Law (Kirchhoff’s Current Law)

The sum of the currents flowing towards a node is equal to the sum of currents flowing away from that node:

$$\sum_{k=1}^{n} I_k = 0$$

From Kirchhoff‘s First Law you obtain the joint resistance R_{ges} of a parallel connection of M resistances:

$$\frac{1}{R_{Ges}} = \sum_{k=1}^{M} \frac{1}{R_k}$$

Kirchhoff’s Second Law (Kirchhoff’s Voltage Law)

The sum of the electrical potential differences around any closed loop is zero:

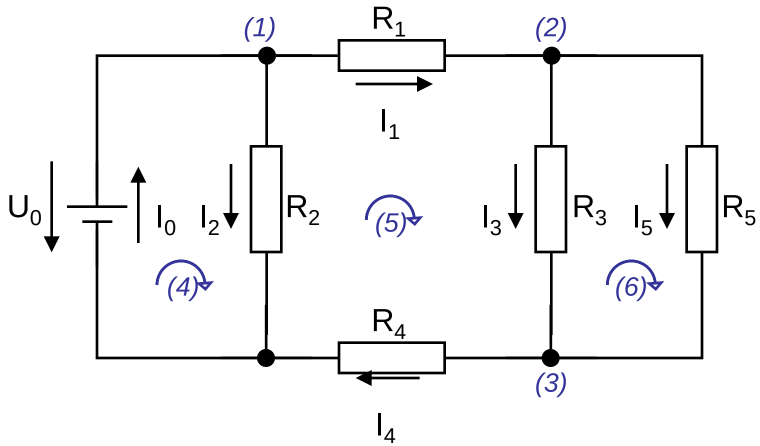
$$\sum_{j=1}^{m} U_j = 0$$

From Kirchhoff‘s Second Law you obtain the joint resistance R_{ges} of a serial connection of M resistances:

$$R_{Ges} = \sum_{k=1}^{M} R_k$$

Electrical Network

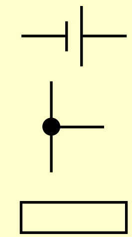
A circuit can be graphically displayed like this:



$R_1 = 120\ \Omega$
$R_2 = 284\ \Omega$
$R_3 = 100\ \Omega$
$R_4 = 20\ \Omega$
$R_5 = 60\ \Omega$
$U_0 = 142$ V

Symbols:

Battery
Branching
Resistance



Linear equation systems are commonly used in many professional areas, e. g. in engineering, economics and finance.

An important application of linear equation systems are current flow considerations in electrical networks.

Ohm‘s and Kirchhoff‘s laws are used to solve the electrical network problem defining a system of linear equations:

$$
\begin{array}{lcccccccc}
(Gl.1) & I_0 & -I_1 & -I_2 & & & & = & 0 \\
(Gl.2) & & I_1 & & -I_3 & & -I_5 & = & 0 \\
(Gl.3) & & & & I_3 & -I_4 & +I_5 & = & 0 \\
(Gl.4) & & & R_2 I_2 & & & & = & U_0 \\
(Gl.5) & & R_1 I_1 & -R_2 I_2 & +R_3 I_3 & +R_4 I_4 & & = & 0 \\
(Gl.6) & & & & -R_3 I_3 & & +R_5 I_5 & = & 0
\end{array}
$$

(Gl.1)–(Gl.3): Kirchhoff’s First Law

(Gl.4)–(Gl.6): Kirchhoff’s Second Law

Ohm’s Law: U = R I

Kirchhoff’s First Law :
$I_1 + I_2 + I_3 + \ldots + I_N = 0$

Kirchhoff’s Second Law :
$U_1 + U_2 + U_3 + \ldots + U_N = 0$

The network problem discussed here is kept very simple to show the general idea.

Analysis of Electrical Circuits

To solve systems of equations, present the information in matrix form:

$[A_{i,j}]\ \{x_i\} = \{b_i\}$

Entering the values of our example we obtain the following system of equations with the current in the circuit as the vector of unknowns:

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 284 & 0 & 0 & 0 \\ 0 & 120 & -284 & 100 & 20 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & -100 & 0 & 60 \end{bmatrix} \cdot \begin{Bmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 142 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Those values of the current in the circuit I_i which satisfy all equations simultaneously are the solution of the linear equation system:

$I_i = \{1.3; 0.8; 0.5; 0.3; 0.8; 0.5\}$ for i = 0, 1, ... 5

Vector of unknowns

$[A_{i,j}]\ \{x_i\} = \{b_i\}$

Vector of absolute terms

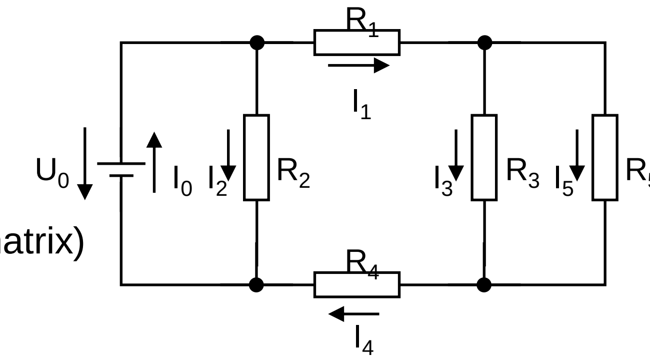
Regular coefficient matrix

Sort the rows in such a way that thererare no zeros on the diagonal of the coefficient matrix.

Final sequence of equations is:
(1) – (2) – (4) – (5) – (3) – (6)

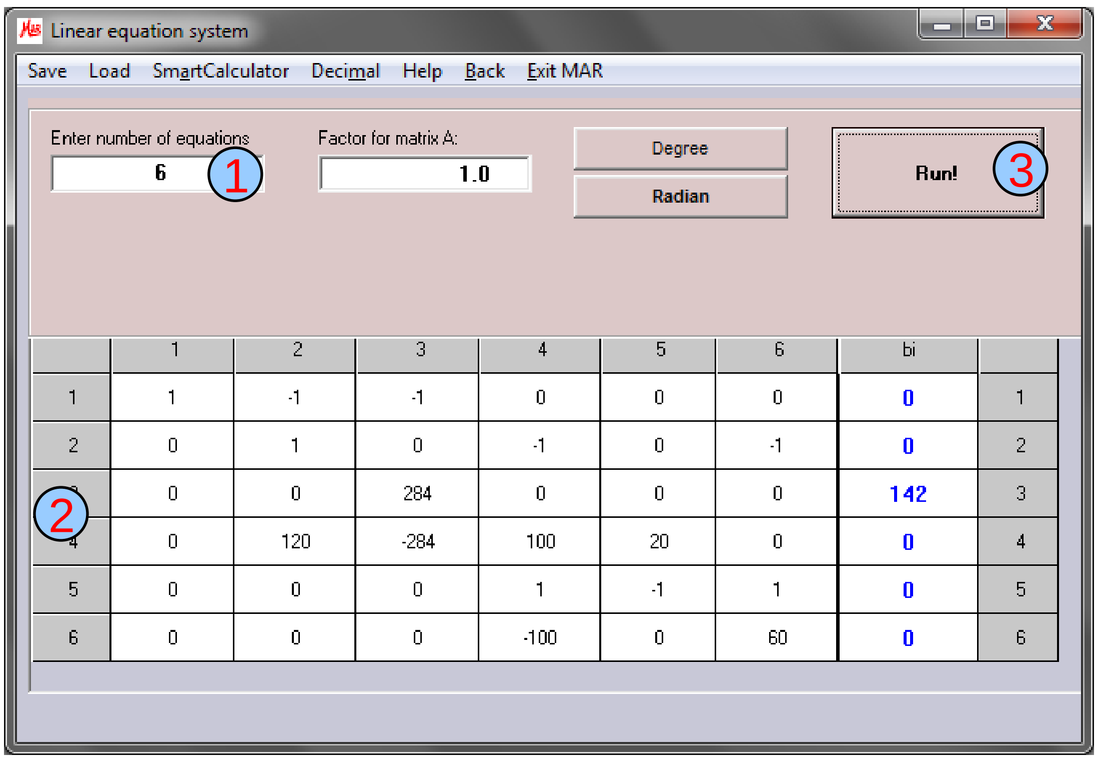
There are three possible outcomes for a linear equation system
- no solution
- a unique solution
- infinite number of solutions

Solution with MAR Plus



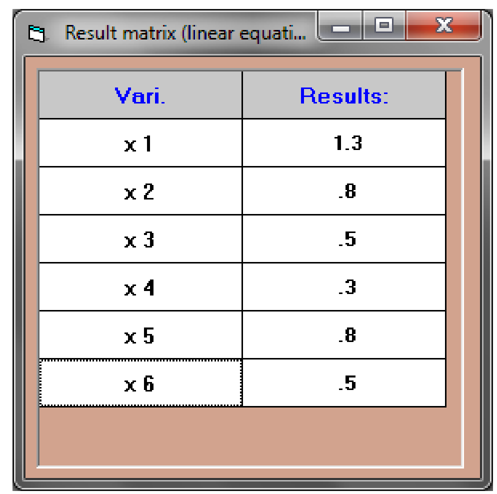
The calculation with MAR Plus is easy and straightforward :

1. Enter the number of equations
2. Enter the coefficients of the matrix and the column vector (augmented matrix)
3. Click on the "Run!" button



	1	2	3	4	5	6	bi	
1	1	-1	-1	0	0	0	0	1
2	0	1	0	-1	0	-1	0	2
3	0	0	284	0	0	0	142	3
4	0	120	-284	100	20	0	0	4
5	0	0	0	1	-1	1	0	5
6	0	0	0	-100	0	60	0	6

The resulting vector gives the values of the current in the circuit:



Vari.	Results:
x 1	1.3
x 2	.8
x 3	.5
x 4	.3
x 5	.8
x 6	.5

$$= \begin{Bmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{Bmatrix}$$